Fixing integer ambiguities for GPS carrier phase time transfer

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Abstract— **GPS is widely used for time and frequency transfers between ground stations. It is well-known that the GPS carrier phase is a much more precise observable than the code and is therefore of great interest. However the carrier phase is ambiguous, only the code can provide the absolute time difference between the two stations.**

Here we propose a rapid method to easily resolve integer ambiguities for accurate time transfer. Using this method, several time transfer results are analyzed for different baselines (from an ultra-short one in common clock configuration to a transatlantic one) and for durations of several days.

Moreover our results will be compared to TWSTFT (Two-Way Satellite Time and Frequency Transfer). Finally this method is deemed to be a reliable way to estimate integer ambiguities.

I. Introduction

GPS time transfer is known to be a convenient and inexpensive way to compare ground clocks. Code measurements provided by dual frequency GPS receivers for instance are quite easy to process. However the performance is limited by the important noise of the code. A classical solution to overcome this is to smooth the code data, using for instance the Vondrak smoothing. Another solution is to use the phase measurements that are much less noisy but their use has some drawbacks.

The phase measurements are ambiguous : they are defined modulo an integer number of wavelengths. The ambiguity value remains constant during a pass when the receiver is correctly locked. A straightforward but not satisfactory method to solve for the ambiguity is to simply adjust also the ambiguities in the global solution. This is of course not a rigorous way to solve for the ambiguities, in particular because it does not take into account their integer nature.

So a rigorous way to carry out time transfer using GPS phase measurements is to solve for the integer ambiguities. Integer blocking was initially performed on double differentiated measurements for geodetic applications. The double difference process has two advantages: first it eliminates directly the clocks from the solution (leading to smaller problems to solve), second, it eliminates common errors in the measurements (for example it eliminates biases due to receiver or emitter).

Different methods have already been developed for integer ambiguity searching like the LAMBDA method [1], but the latter is more focused on real-time data processing (especially for precise positioning). Here we propose a simpler method that has the advantage of simplicity and is well adapted to offline least-square analysis.

The presented approach uses a combination of all-in-view and common-view techniques and single differences for code and phase measurements. The single difference process allows the elimination of common errors similarly to the double differences methods but without eliminating the receiver clocks.

Advantages of integer ambiguity resolution have already been discussed [2]. It is generally acknowledged, and we could verify it indeed, that the integer ambiguity resolution allows to stabilize the GPS solution, making it more robust to potential mis-modellings for instance [2,3].

Several time transfer results using this method are given in this paper and whenever possible compared to TWSTFT.

II. Ambiguity resolution method

A. Methodology

For a given baseline, the general methodology to estimate the time offset between the two ground stations with the integer ambiguities resolved is as follows :

- pre-processing of the measurements of both stations, computation of the geometry and of the partial derivatives
- absolute positioning (aka all-in-view technique) for both stations (using IGS precise ephemeris and clocks)
- relative solution (aka common-view technique) with estimation of the integer ambiguities.

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The absolute positioning solution uses floating ambiguities solution on the iono-free code and phase combinations. The troposphere and coordinates (if necessary) are adjusted, together with the receiver clock relative to the constellation solution time.

The relative solution uses single differences in order to have integer ambiguities properties. These ambiguities are identified together with receiver clocks differences. In this solution the coordinate and troposphere delays are fixed to the absolute positioning results. This approach is similar to the improvement suggested in [3] (include the double difference integer ambiguity constraint in the equations).

B. Measurements pre-processing

The pre-processing is carried out by sequential finite differences on the measurements. This allows to define the continuous passes for phase measurements on the two frequencies. Floating phase ambiguities N_1 and N_2 are estimated on each pass. The ambiguities N_1 and N_2 are defined as:

$$e_p = \frac{P_1 - P_2}{1-\gamma}$$

$$N_1 = -\left\langle L_1 - \frac{P_1 - 2e_p}{\lambda_1} \right\rangle$$

$$N_2 = -\left\langle L_2 - \frac{P_2 - 2\gamma e_p}{\lambda_2} \right\rangle$$

where γ is the ratio f_1^2/f_2^2 and e_p is the estimated ionosphere delay using the P code measurements.

By combining these data, the iono-free code and phase combinations can be constructed for the absolute positioning:

$$P_c = \frac{P_2 - \gamma P_1}{1-\gamma} \quad \text{and} \quad Q_c = \frac{\lambda_2(L_2 + N_2) - \gamma\lambda_1(L_1 + N_1)}{1-\gamma}$$

For the measurement modelling and partial derivatives relative to station coordinates and vertical troposphere delay, it is necessary to have the geometry of the problem. For the position of the GPS satellites, we first estimate the emission epoch using a code measurement (C1 for instance) and the associated clock correction h_{emi}. Then, the IGS ephemeris are interpolated at this emission date. The model equations are:

$$P_c = D_{geo} + D_{tropo} + (h_{rec} - h_{emi})$$

$$Q_c = D_{geo} + D_{tropo} + D_{wind} + (h_{rec} - h_{emi}) - R_c$$

where D_{geo} is the geometrical distance between the iono-free phase centers of the emitter and the receiver antennas, D_{wind} corresponds the effect of the relative phase rotation between the antennas of the emitter and receiver (so-called wind-up effect) and D_{tropo} is the troposphere propagation delay. R_c is a floating value, which is close to 0, due to the previous estimations of N_1 and N_2 which are applied in the iono-free phase measurement construction. h_{rec} is the receiver clock offset (expressed in meters).

C. Absolute positioning

This step consists in estimating the receiver clock with respect to the reference time (here IGST) at each epoch, with identification of a floating ambiguity per pass, of the three station coordinates and of the vertical troposphere delay. There is no constraint on the coordinates or on the clock of the station. Vertical troposphere delay is defined every two hours with a linear evolution in-between. A constraint is applied on the difference between two successive values.

D. Relative solution

With the outputs from absolute positioning, we can form the single differences, represented here by the operator Δ. The troposphere is kept from the absolute positioning, which provides a better observability since all satellites in view are used.

The integer widelane ambiguity $\Delta N_w = \Delta(N_2 - N_1)$ is identified on single differences. The iono-free phase combination can then be constructed as:

$$\Delta Q_c = \frac{\lambda_2(\Delta L_2 + \Delta N_w) - \gamma\lambda_1\Delta L_1}{1-\gamma}$$

The single difference equations to solve are now:

$$\Delta P_c = \Delta(D_{geo} + D_{tropo}) + \Delta h_{rec}$$

$$\Delta Q_c = \Delta(D_{geo} + D_{tropo} + D_{wind}) + \Delta h_{rec} - \lambda_c N_c$$

where the wavelength $\lambda_c = \frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}$ corresponds to the effect of the integer *L1* cycles (which are not known yet) on the iono-free phase combination, and with Δh_{rec} the clock difference between the two receivers and N_c an integer correction for each single difference pass.

The geometrical modelling is to be precise enough for N_c to appear as an integer value. This is very easy on short baselines. Difficulties may arise for longer baselines (typically more than 2000 km).

The computation of the ambiguities is done sequentially by adjusting each integer ambiguity on the preceding adjusted ones, together with clock estimation epoch by epoch (using only the iono-free phase equation). An ambiguous clock h_{phase} is obtained (after estimations of the integer N_c values) :

$$h_{phase} = \left\langle \Delta Q_c - \Delta(D_{geo} + D_{tropo} + D_{wind}) + \lambda_c N_c \right\rangle_{epoch}$$

(mean epoch by epoch)

This h_{phase} clock is defined modulo an integer number of λ_c. It is then possible to estimate an overall offset to be applied on this ambiguous clock to obtain the complete clock, using the iono-free code equations:

$$dh = \left\langle \Delta P_c - \Delta(D_{geo} + D_{tropo}) - h_{phase} \right\rangle$$

(mean for all epochs)

dh is not precisely estimated due to the code noise and therefore, if one wants to reconstruct continuous solutions from overlapping solutions, it is interesting to separate this correction from the integer clock h_{phase}, because the differences between these h_{phase} clocks remain integer numbers of λ_c.

With the equations used above for the modelling, the *dh* value is an integer number of cycles. In fact, the processing on long uninterrupted data sets shows that there is probably a slowly varying 'phase-code' bias term due to the receivers, which must be taken into account for very precise applications. This term is not observable in standard double difference processing, as it is common between all measurements performed at the same epoch, and also appears between code and phase measurements.

III. TIME TRANSFER RESULTS

A. Ultra-short baseline, common clock

A specific test has been conducted in order to validate the methodology. The test configuration has two receivers (here an Ashtech Z-12T and a Septentrio PolaRx2) connected on the same reference clock, with different antennas. The antennas are very close (a few meters).

The advantage of this configuration is that the receiver clock difference (which is a constant bias due to the various delays in the experimental setup) can be estimated without troposphere and ionosphere effects. This allows independent estimations on each frequency, without adjusting any vertical troposphere delay values.

The following figures show the time histories and the Allan deviations obtained for the receiver clocks differences for L1, L2, and also for the estimation using the complete iono-free computation (where troposphere delays are estimated independently on the two receivers data sets).

The residuals correspond to the formula $\Delta L_1 - \frac{\Delta D_{geo}}{\lambda_1} + N_1$ for frequency 1 (similar expression for L2), plotted with one colour per single difference pass, independently for each frequency.

The L1 clock has the expected slope -1 corresponding to the phase measurement noise, and has no drift at all.

But the L2 clock is very different, and an additional noise appears on the result (see figure 1), which is higher than the measurement noise and produces in the residuals a behaviour corresponding to a common noise between all measurements at the same epoch. This means that the clock issued from the iono-free combination will show a behaviour corresponding to the combination of the L1 and L2 clocks with respectively the coefficients $\frac{-\gamma}{1-\gamma} \sim 2.5$ and $\frac{1}{1-\gamma} \sim -1.5$.

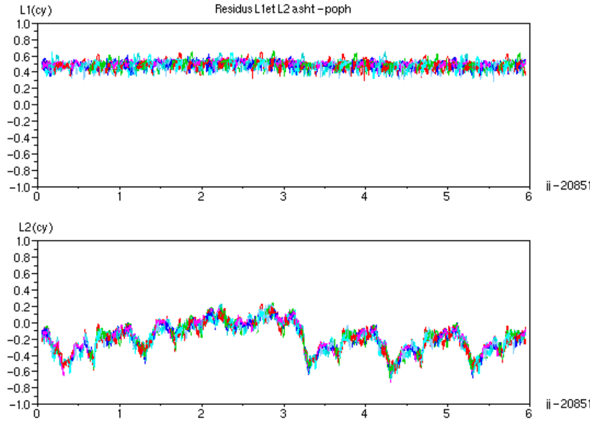

Figure 1. L1 and L2 residuals (receiver clock not removed) (abcissa in days)

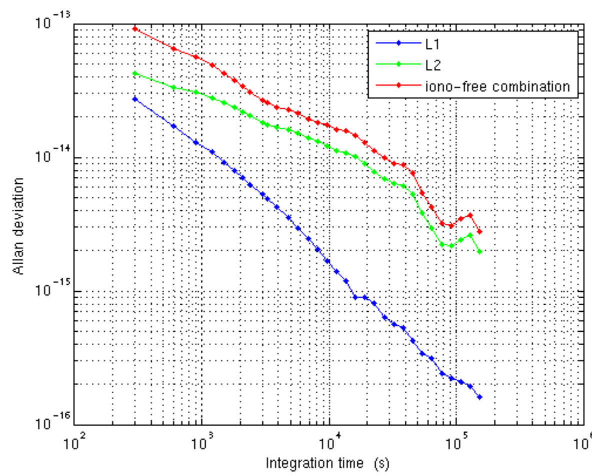

Figure 2. Allan deviation for L1, L2 and the complete solution

The clock estimated with the complete method (red curve) has exactly this characteristic.

By comparing these results with a third receiver located close to these two receivers (IGS [4] station TLSE), but not connected to the same clock, it is possible to show that the origin of this behaviour is the L2 measurement of the Septentrio receiver). This has not been explained yet (receiver configuration problem…).

B. Continental baseline

We considered the baseline OPMT/PTBB which is about 700 km. OPMT is an IGS station located in Paris Observatory, it is driven by a Hydrogen Maser which is compared to UTC(OP) on an hourly basis. The receiver is an Ashtech Z-12T. The station is calibrated so that we can relate the measurements to UTC(OP). PTBB is also an IGS station located in PTB (Braunschweig, Germany), it is directly driven by UTC(PTB) and the calibration values are indicated in the header of the CGGTTS files. Calibration delays for both stations have been used for the computation of GPS CP (carrier phase) using integer ambiguity resolution. Therefore, a GPS time transfer between these stations allows to compute

UTC(OP) – UTC(PTB). This offset is also computed by BIPM using TWSTFT and GPS P3 CV (Common-View) and AV (All-in-View) using Vondrak smoothing. The BIPM data have been retrieved from their ftp site [5].

In Figure 3, we compare the TWSTFT and GPS P3 CV from BIPM data to our GPS CP time transfer using integer ambiguity resolution:

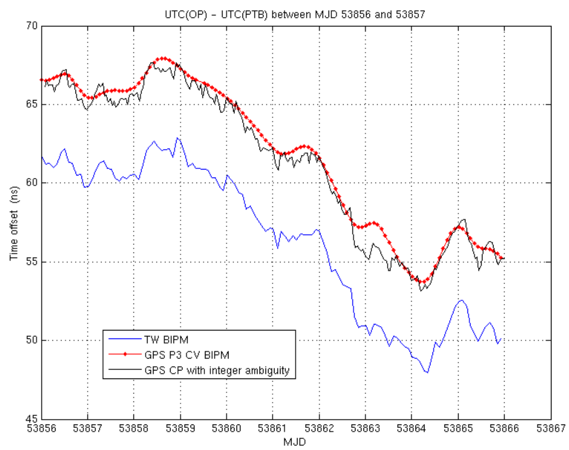

Figure 3. Time offset UTC(OP) – UTC(PTB) computed by three different techniques

We can observe a very good agreement between the TWSTFT and GPS CP using integer ambiguity resolution. Short term variations are very similar with these two techniques, while they are invisible with GPS P3 CV and AV with Vondrak smoothing. We can also notice that the GPS carrier phase using integer ambiguity resolution has an absolute value that fully agrees with BIPM GPS P3 CV. Note that the BIPM GPS P3 AV is very close to the CV and has therefore not been plotted here.

Figure 4 is the difference between the TWSTFT and GPS CP using integer ambiguity resolution (mean removed) :

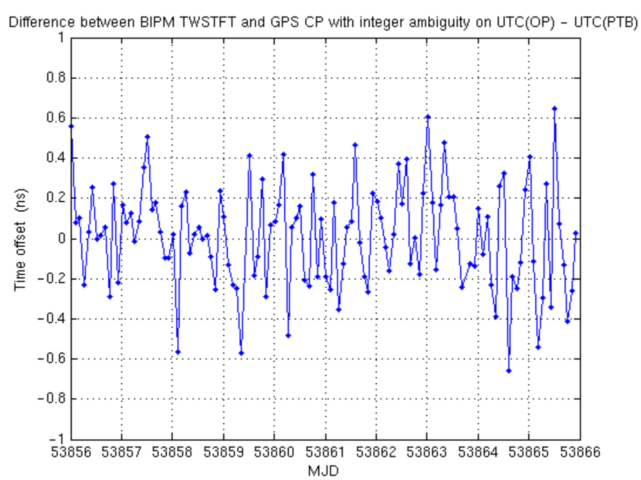

Figure 4. Difference between TWSTFT and CNES GPS carrier phase with integer ambiguity on the time offset UTC(OP) – UTC(PTB)

The standard deviation of the difference is 0.25 ns.

Figure 5 shows the stability of UTC(OP) – UTC(PTB) computed with TWSTFT, GPS P3 CV from BIPM data and CNES GPS carrier phase (CP) with integer ambiguity resolution.

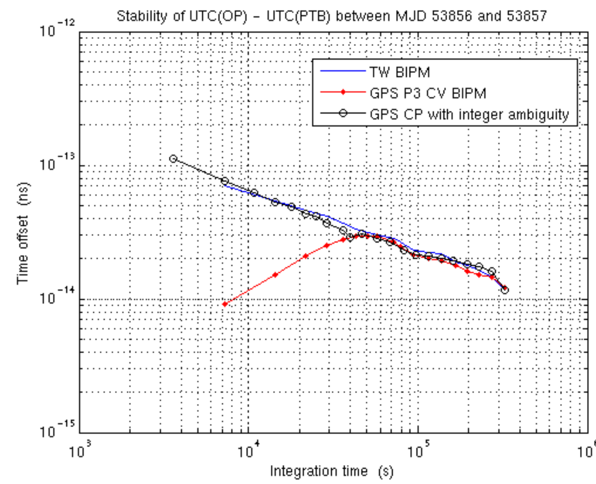

Figure 5. Allan deviation of the time offset UTC(OP) – UTC(PTB) for the three different techniques

The stabilities of the time offset UTC(OP) – UTC(PTB) TWSTFT and GPS CP with integer ambiguity agree very well. The apparent good short term stability of the BIPM GPS P3 CV is only due to the smoothing and is not deemed to be representative of the clock difference behaviour.

C. *Transatlantic baseline*

We considered the baseline USN3/PTBB which is about 6300 km. USN3 is an IGS station located at US Naval Observatory, it is driven by UTC(USNO) with no corrections to apply (see USN3 IGS log file).

Similarly, a GPS time transfer between these stations allows to compute UTC(USNO) – UTC(PTB). And similarly the results can be compared to the BIPM data. However, for our integer ambiguity resolution, we had to use two bridge stations (STJO and REYK) because of the length of the baseline. For the initial baseline, the direct ambiguity solution is not efficient, because the modelling errors are more important (troposphere modelling at low elevations for example), the single difference passes are shorter, and there are not enough simultaneous passes. With the two other stations it is possible to solve on three shorter baselines (~2500 km).

First the tree baselines are processed independently (but with the same absolute modelling parameters), giving three estimates for the following clock relations (6 days solution). Table 1 summarizes the results.

TABLE I. N1 AMBIGUITIES ON THE THREE BASELINES

Baseline	***Length (km)***	***Total N1***	***Integer N1 (%)***	***Clock***
PTBB-REYK	2265	231	191 (83 %)	H12
REYK-STJO	2597	230	199 (87 %)	H23
STJO-USN3	2179	152	145 (95 %)	H34

During the identification of the integer ambiguities values, some passes have been rejected due to inconsistent residuals behaviour (threshold used: 0.25 cycle on the overlap between two passes).

Then the clock difference for the base line PTBB-USN3 is obtained by H12+H23+H34. Due to the fact that the models used are the same for a station contribution on two different baselines, there are no biases accumulations due for example to different troposphere estimations on the baselines.

Figure 6 shows the time histories obtained for two overlapping 6 days solutions. For each arc, the integer and floating solutions corrected with the rounded bias are shown. As recalled in [2,6], floating solutions may exhibit important drifts, which are not present in the integer solutions. Also here, due to the use of rounded biases, the integer clocks differ exactly with one cycle bias as it is visible on the central day. The reconstruction of a complete clock is thus straightforward.

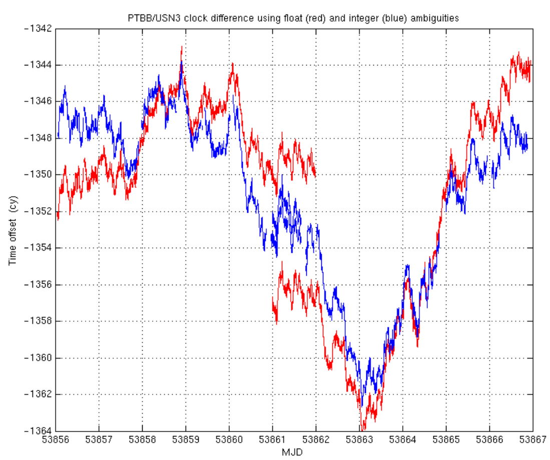

Figure 6. Time history for integer (blue) and floating (red) solutions (units in cycles)

In the following Figure, we compare the 3 BIPM techniques to our GPS carrier phase time transfer using integer ambiguity resolution:

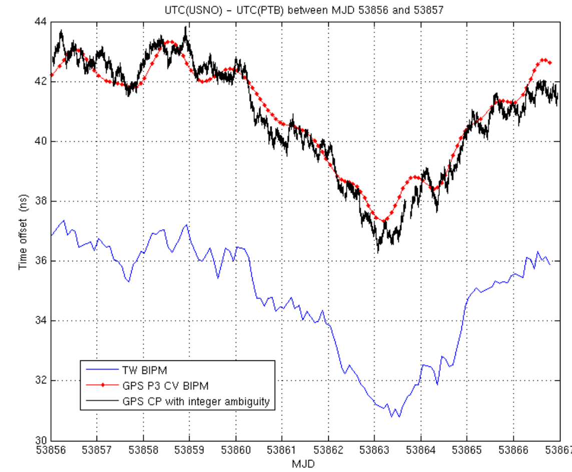

Figure 7. Time offset UTC(USNO) – UTC(PTB) computed by three different techniques

Here again, we have a very good agreement between the TWSTFT and GPS carrier phase using integer ambiguity resolution. Short term variations are similar with these two techniques, while they are invisible with GPS P3 CV and AV with Vondrak smoothing. We can also notice that the GPS carrier phase using integer ambiguity resolution has an absolute value that fully agrees with BIPM GPS P3 CV. Note that the BIPM GPS P3 AV is very close to the CV and has therefore not been plotted here.

The Figure below is the difference between the TWSTFT and GPS carrier phase using integer ambiguity resolution on this baseline with the mean removed:

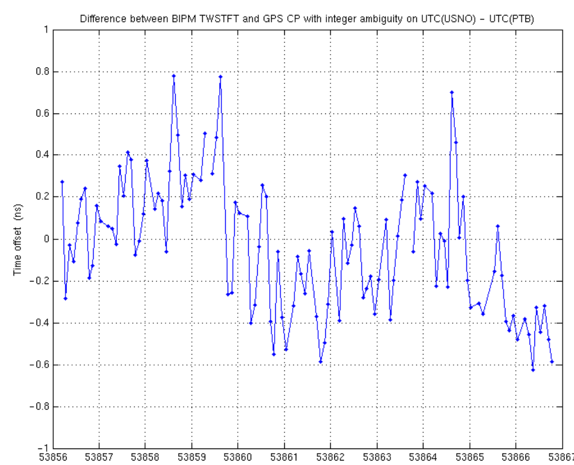

Figure 8. Difference between TWSTFT and CNES GPS carrier phase with integer ambiguity on the time offset UTC(USNO) – UTC(PTB)

The standard deviation of the difference is 0.3 ns. This result is better than a similar comparison [3] based on CODE products on the same baseline but on a much longer duration.

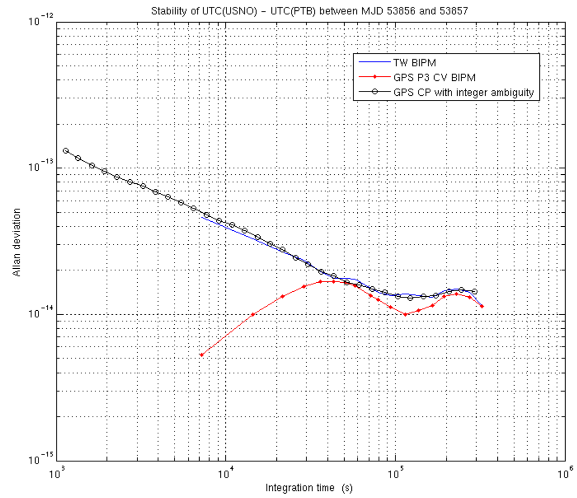

Figure 9. Allan deviation of the time offset UTC(USNO) – UTC(PTB) for the three different techniques

IV. Conclusion

A simple method for the resolution of the integer ambiguities has been presented. This approach has been tested on different cases and its results successfully compared to the TWSTFT on the baselines OP-PTB and USNO-PTB.

Agreement with TWSTFT at the level of 0.3 ns (standard deviation on 11 days) has been achieved even on transatlantic baselines. It would be of particular interest to renew such a comparison on a longer duration in order to confirm this good agreement and to investigate possible seasonal effects.

Due to the difficulty to solve directly on very long baselines, it was necessary to add stations, in order to solve only for shorter baselines. The next step is to improve the processing to solve directly in an integer solution the complete set of baselines corresponding to a network of stations.

The level of precision which has been obtained now allows the observation of specific receiver delays ('code-phase' delay). Further studies are necessary to characterise the associated new models.

Acknowledgment

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